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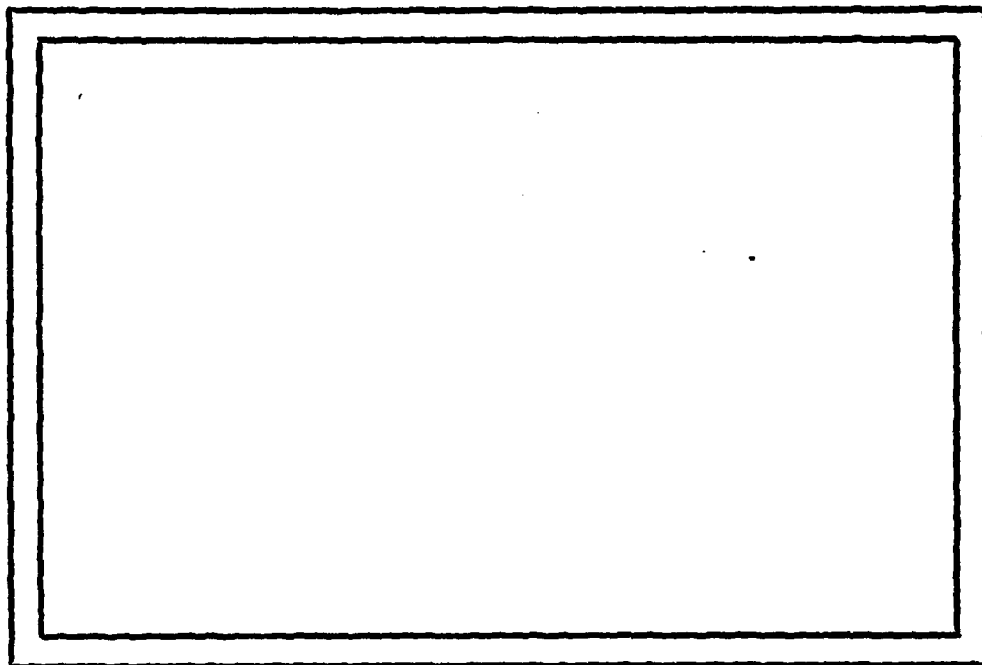
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On the Optimality of
Semidynamic Routing Schemes

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Abstract

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Key Words: resource allocation,   performance measurement

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1. Introduction

The routing schemes reported in the literature can be classified into two basic categories, Dynamic (adaptive) and Static[1]. The dynamic schemes require information about the state at the destination which is obtained through explicit mechanisms which may involve significant overhead. The accuracy (i.e., currentness) of such information depends on the communication delays. Further, such algorithms rarely lend themselves to analysis. The static algorithms are based on the information available at the time of designing the system and do not change as a function of arrivals or loads. An example of such routing are stochastic rules [Schw 77].

Recently a new class of algorithms under the name "Semidynamic" were introduced [AT 80] [Yum 79] [EVW 79] where, while no information from the destination is sought, the controller retains some information from the past and uses it in making its decisions. Extensive analysis of stochastic rules is available in the literature, and the ways of determining the optimal stochastic rule for different speed servers is known [Schw 77]. The study of semidynamic rules is only in its infancy. In [EVW 79] the authors show that for two equal speed servers the deterministic rule of routing to them alternately is the best. YUM [Yum 79] has shown that a deterministic rule based on the best stochastic rule gives better delay performance than the stochastic rule.

Based on a recently developed technique [AT 80], in this paper we show that the deterministic techniques based on the best stochastic rules are not necessarily optimal.

2. Best Stochastic and Semidynamic Rules

Consider an example where a controller (C) routes the arriving

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[1]There are many ways of classifying routing techniques, [McQu 77] [Gall 77] [DT 79].

customers to one of the many nodes (Ni); see figure 1.

$$\begin{array}{c} \rangle \boxed{N_1} \\ \\ \longrightarrow \boxed{C} \quad \Big| \qquad \rangle \boxed{N_2} \qquad \rangle \\ \vdots \\ \rangle \; N_n \end{array}$$
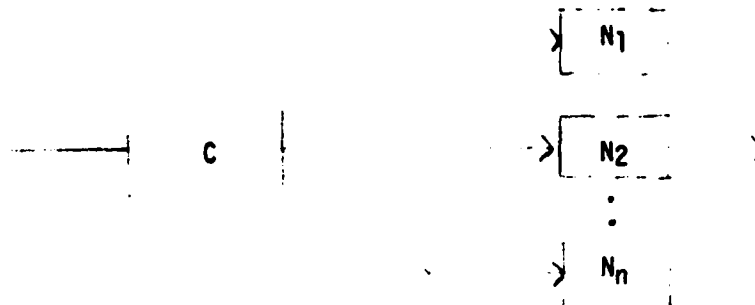
Figure 1

The optimal rule for a given class is the rule of that class (for example, Stochastic rules) which minimizes the mean delay in the system[2]. Assume that the external arrival rate is λ and the processing rate of node i is M_i. The interarrival time and the processing time are each exponentially distributed. An optimal routing probability to node i, P_i, can be obtained [BC 74].

A deterministic (semidynamic) rule corresponding to a stochastic rule is based on a deterministic routing sequence   S:

$$S = \left\{ S_1, S_2, \ldots S_m \right\}$$

with $S_i = k$ meaning a routing decision in favor of node k for the ith incoming message. For any subsequence of length k, let $D(i/k)$ be the number of i-decisions, i = 1,2,...n. Sequence S is constructed in such

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2 The problem can easily be extended to a network and in that case the routing problem is similar to the Multi-commodity Flow problem in Network Flow Theory. In this paper, however, we restrict ourself to simple single hop strategies.

a way that $\underline{D(i/k)}$, the fraction of messages routed to Ni in the total of
$\quad\quad k$

k messages, is as close to Pi as possible for all k [Yum 79]. As an

example,consider n=2, P_1=2/3, and P_2=1/3. S is given by:

$$S = \left\{ [1,1,2] \right\}$$

where [·] means sequence inside is to be repeated. Such routing schemes

are also referred as cyclic routing where the cycle is 1,1,2 [AT 80].

3. Optimality of Semidynamic Rules

Semidynamic rules are not only easy to implement but also give better

performance than their stochastic parents[Yum 79]. Once the class of

admissible rules is expanded to semidynamic rules, however, we can obtain

a sequence that gives better performance than the one based on the best

stochastic rule.

Consider an example where we have two nodes with the following sequence:

$$S = \left\{ [\underbrace{1\ldots1}_{n_{11}} \; \underbrace{2\ldots2}_{n_{21}} \; \underbrace{111\ldots1}_{n_{12}} \; \underbrace{2\ldots2}_{n_{22}} \; \ldots \ldots \; \underbrace{1\ldots1}_{n_{1k}} \; \underbrace{2\ldots2}_{n_{2k}}] \right\}$$

where $n_{ij} > 0$. Note that $P1 = \sum_{i=1}^{k} n_{1i} \bigg/ \sum_{i=1}^{k}(n_{1i} + n_{2i})$

and $P2 = \sum_{i=1}^{k} n_{2i} \bigg/ \sum_{i=1}^{k}(n_{1i} + n_{2i})$

Let us look at the node N_1. The interarrival time distribution

this node is given by

$$A = [E_{n_{2k}}\underbrace{E_1\ldots E_1}_{n_{11}-1}, \; E_{n_{21}}, \underbrace{E_1\ldots E_1}_{n_{12}-1}, E_{n_{22}}, \underbrace{E_1\ldots E_1}_{n_{31}-1}, \ldots, E_{n_{2k-1}}, \underbrace{E_1\ldots E_1}_{n_{1k}}]$$

where E_i denotes an i-fold convolution of exponential distributions with

the same rate (the arrival rate). Thus, node N is a $G^n/M/1$ queue where

the arrival is from different distributions of a cycle length n. Solutions

to such a queue are presented elsewhere [AT 80] and we will use them here to

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show the non-optimality of semidynamic rules based on optimal stochastic rules.

Let $M_1 = 4$, $M_2 = 1$ and $\lambda = 3$ (requests per unit time). From [BC 74] $P_1 = 8/9$ and P2 = 1/9, and the mean time delay w* is given by w* = 5/6 time units. The corresponding semidynamic rule is given by the following sequence:

$$(8,1) = \left\{ [1,1,1,1,1,1,1,1,2] \right\}$$

where (i,j) denotes a sequence with i jobs going to node 1 and j to node 2. Thus the interarrival time distributions to server 1 are E_2 followed by seven E_1's and to server 2 is E_8. The mean time delay for the $(8,1)$ rule is given in Table 1. This table also gives the waiting time for $(7,1)$, $(6,1)$, $(5,1)$ and $(4,1)$. Clearly the mean time delay for $(8,1)$ is less than w* but the corresponding delay for $(7,1)$ is less than that for $(8,1)$. From the figures in the table we note that $(6,1)$ gives the minimum mean delay. However, the optimum deterministic semidynamic routing strategy may involve a complex sequence sending a fraction of the incoming jobs between one out of 8 and one out of 6.

Table 1 $\lambda = 3.0$ $M_1 = 4.0$
                                      $M_2 = 1.0$

Scheme	Mean Delay
Probabilistic	0.8333
(8,1)	0.7642
(7,1)	0.7535
(6,1)	0.7469
(5,1)	0.7532
(4,1)	0.8018

Further result from the routing sequences with proportions between one out of 8 and one out of 6 is given in Table 2.

Table 2

Proportion	Scheme	Mean delay
2/13	{(6,1)(5,1)}	.7478
3/20	{(6,1)(6,1)(5,1)}	.7472
4/27	{(6,1)(6,1)(6,1)(5,1)}	.7470
1/7	{(6,1)}	.7469
3/22	{(6,1)(6,1)(7,1)}	.7485
2/15	{(6,1)(7,1)}	.7495

Note that the minimum delay occurs around one out of 7 proportion. However, obtaining exact optimal sequence is still an open problem.

4. **Conclusions**

The results presented in this paper show than an optimal semidynamic deterministic routing scheme cannot be constructed simply by using the optimal stochastic proportions. This observation is made from a simple counterexample. The approach to obtain the optimal deterministic sequences needs further investigation.

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